

Lecture 7 «Mass-exchange processes. Phase balance. Material balance of mass-exchange processes. Main equation of a mass transfer. Average driving force of process of a mass transfer»

Aim: Formulate the mass-exchange processes. Characterize the phase balance. Describe the material balance of mass-exchange processes. Write the main equation of a mass transfer. Characterize the average driving force of process of a mass transfer.

Lecture summary: Basics of mass transfer. Technological processes, the rate of which is determined by the rate of transfer of a substance (mass) from one phase to another, are called *mass-exchange processes*. Such processes include: absorption, adsorption, rectification, extraction, drying, crystallization.

The rate of these processes is determined by the rate of diffusion. Processes in which the transition of substance from one phase to another occurs by diffusion are called mass transfer processes. In the processes of mass transfer, two phases participate in which the third substance is distributed. The phases are carriers of the substance to be distributed and do not participate directly in the mass transfer process.

Phase equilibrium

The transition of substance from one phase to another occurs in the absence of equilibrium between the phases. At equilibrium, a definite relationship is established between the limiting, or equilibrium, concentrations of the distributed substance in the phases for the given temperature and pressure at which the mass transfer process takes place.

In the most general form, the relationship between the concentrations of a distributed substance in phases at equilibrium is expressed by the dependence:

$$y^* = f(x) \quad (1)$$

or

$$x^* = f(y) \quad (2)$$

where x – the content of the substance distributed in one phase, y^* – the equilibrium concentration of this substance in the other phase and vice versa (equation (2)).

The equilibrium conditions (1) and (2) allow us to determine the direction of the process. If the working concentration of the distributed substance in this phase is above equilibrium, then it will leave this phase in a different phase.

The equilibrium between the phases can be represented graphically on the $y - x$ diagram (Fig. 1) [2,3].

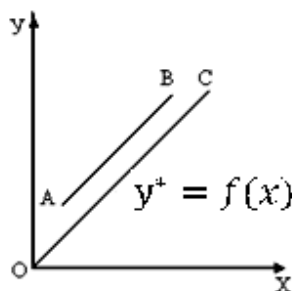


Fig. 1. Diagram $y-x$:
OC – equilibrium line;
AB – working line

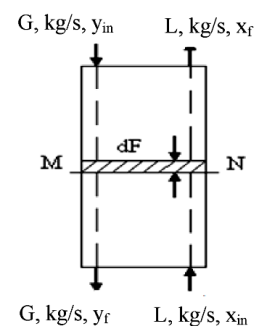


Fig. 2. To the derivation of equation
line of working concentration

Material balance of mass-exchange processes

Diffusion (mass exchange) processes, as a rule, are carried out in countercurrent apparatuses, where the phases participating in the mass exchange flow towards each other. Therefore, to derive the equation for the material balance of mass exchange processes, let us consider the scheme of a mass-exchange apparatus operating in the ideal-displacement mode in countercurrent phases (Fig. 2).

We denote the weight velocities of the phases of liquid L and gas G along the interface in kilograms per hour. The content of the distributed component in them will be denoted in kilograms per kilogram of phase: in phase L – through x and in phase G – through y .

Let us assume that the working concentration of the distributed component is above its equilibrium concentration $y > y^*$, and therefore the component will pass from phase G to phase L .

The phases are carriers of the substance being distributed and do not participate in the mass exchange process. For an infinitely small surface element dF of the phase contact, the material balance with respect to the component distributed between the phases is expressed by the differential equation

$$dM = -Gdy = Ldx \quad (3)$$

Integrating the equation within the given limits of the concentrations of the distributed substance from y_{in} to y_f and from x_{in} to x_f

$$M = -G \int_{y_{in}}^{y_f} dy = L \int_{x_{in}}^{x_f} dx \quad (4)$$

or

$$M = G(y_{in} - y_f) = L(x_f - x_{in}) \quad (5)$$

we obtain the equation of the material balance of mass exchange for the entire surface of phase contact in the apparatus under consideration.

From the equation a the relationships between the weight flows of phases are found

$$L = G \frac{y_{in} - y_f}{x_f - x_{in}} \quad (6)$$

and

$$G = L \frac{x_f - x_{in}}{y_{in} - y_f} \quad (7)$$

and specific consumption of solvent

$$l = \frac{L}{G} = \frac{y_{in} - y_f}{x_f - x_{in}} \quad (8)$$

For any arbitrarily taken section of the apparatus below the line MN (Fig. 2) with the phase concentration y and x , integrating the equation of material balance (3) in the range from y to y_f and from x_{in} to x , we obtain

$$-G \int_y^{y_f} dy = L \int_{x_{in}}^x dx \quad (9)$$

we get

$$G(y - y_f) = L(x - x_{in}) \quad (10)$$

the equation of material balance for a part of the apparatus (below MN).

From the equation (10) we find

$$y = \frac{L}{G}x + \left(y_f - \frac{L}{G}x_{in}\right) \quad (11)$$

This equation is called the equation of the working line of the mass exchange process. It expresses the relationship between the nonequilibrium compositions of the phases y , x in any section of the apparatus.

The quantities G , L , y_f , x_{in} are known and are constant, therefore, introducing the notation $\frac{L}{G} = A$ and $y_f - \frac{L}{G}x_{in} = B$, we find

$$y = Ax + B \quad (12)$$

This equation of the straight line, from which it follows that the concentrations of the substance distributed in the phases G and L are connected by a linear relationship [2,3].

Basic equation of mass transfer

The basic law of mass transfer can be formulated on the basis of the general kinetic laws of chemical-technological processes. The speed of the process is equal to the driving force divided by the resistance:

$$\frac{dM}{dF d\tau} = \frac{\Delta}{R} \quad (13)$$

where dM – the amount of substance passing from one phase to another, kg/s; dF – phase contact surface, m²; $d\tau$ – the time, s; Δ – the driving force of the mass transfer process; R – the resistance.

Denoting the inverse of the resistance $1/R$ by K and referring the amount of substance transferred from one phase to another dM to a unit time, equation (13) can be rewritten as follows:

$$M = K dF \Delta, \quad (14)$$

where K – the mass transfer coefficient.

Equation (14) is called the basic mass transfer equation. For the entire surface of the phase contact F , equation (14) is written

$$M = KF\Delta \quad (15)$$

Thus, the amount of substance M , passing from one phase to another in a unit of time, is proportional to the contact surface of the phases F and the driving force Δ .

The dimension of the mass transfer coefficient is determined from equation (15):

$$[K] = \left[\frac{M}{F\Delta} \right] = \left[\frac{kg/s}{m^2 \cdot (u.driv.force)} \right] \quad (16)$$

The mass transfer coefficient is the amount of a substance passing from one phase to another per unit time through a unit of the phase contact surface at a driving force equal to unit.

Average moving force of the process of mass transfer

The driving force Δ can be expressed in any units used to express the phase composition. The driving force of process Δ can be expressed through concentrations in one of the phases:

$$M = K_y(y - y^*)F, \quad (17)$$

$$M = K_x(x^* - x)F, \quad (18)$$

where y, x – the working concentrations of the component being distributed in the gas and liquid phases, respectively; y^*, x^* – the equilibrium concentrations.

If the working and equilibrium concentrations of the substance to be distributed are expressed in terms of relative weight compositions (kg/kg), the mass transfer coefficient will be:

$$[K] = \left[\frac{M}{F\Delta} \right] = \left[\frac{kg/s}{m^2 \cdot kg/kg} \right] = \left[\frac{kg}{m^2 \cdot s} \right] \quad (19)$$

When the driving force is expressed in terms of the difference in partial pressures, $\Delta p = p - p^*$, N/m²:

$$[K] = \left[\frac{M}{F\Delta} \right] = \left[\frac{kg/s}{m^2 \cdot N/m^2} \right] = \left[\frac{kg \cdot s^2 \cdot m^2}{s \cdot m^2 \cdot kg \cdot m} \right] = \left[\frac{s}{m} \right] \quad (20)$$

If the driving force of the process is expressed in terms of the difference in volume concentrations (kg/m³), then the dimension of the mass transfer coefficient will be [2, 3]:

$$[K] = \left[\frac{M}{F\Delta} \right] = \left[\frac{kg/s}{m^2 \cdot kg/m^3} \right] = \left[\frac{m}{s} \right] \quad (21)$$

The driving force varies with the change in working concentrations. Therefore, for the entire process of mass transfer, which takes place within the limits of the change in concentrations from the initial to the final, the average driving force is determined. The expression for the average driving force depends on whether the equilibrium line (other things being equal) is a curve or a straight line.

To determine the average driving force, we consider the process taking place in a countercurrent column apparatus under the following conditions: 1) the $y^* = f(x)$ dependence is a curve; 2) the phase consumptions are constant ($G = \text{const}$, $L = \text{const}$), i.e. the working line is straight; 3) the mass transfer coefficients do not change ($K_x = \text{const}$, $K_y = \text{const}$) along the height of the apparatus (Fig. 2).

In our case, $y > y^*$, therefore, the substance to be distributed passes from phase G to phase L . For the element of the phase contact surface, dF , the driving force can be expressed by the difference in working and equilibrium concentrations $\Delta y = y - y^*$. The amount of substance will pass from phase G to phase L on the surface element dF equal to dM , which will be determined in accordance with the basic mass transfer equation:

$$dM = K_y(y - y^*)dF \quad (22)$$

The same amount of substance can be expressed as

$$dM = dM = G(-dy) \quad (23)$$

Comparing equations (22) and (23) and solving with respect to dF , we find

$$dF = -\frac{Gdy}{K_y(y-y^*)} = -\frac{G}{K_y} \cdot \frac{dy}{(y-y^*)} \quad (24)$$

Integrating equation (24) in the range from 0 to F and from y_{in} to y_f , we obtain

$$\int_0^F dF = -\frac{G}{K_y} \int_{y_{in}}^{y_f} \frac{dy}{y-y^*} \quad (25)$$

or

$$F = -\frac{G}{K_y} \int_{y_{in}}^{y_f} \frac{dy}{y-y^*} \quad (26)$$

For the entire surface of the phase contact F , based on the basic mass transfer equation and the material balance equation, we can write:

$$M = G(y_{in} - y_f) = K_y F \Delta y_m \quad (27)$$

where Δy_m – the average driving force.

From equation (27), the amount of phase G can be expressed as:

$$G = \frac{M}{y_{in} - y_f} \quad (28)$$

We substitute the found value of G into equation (26) and obtain:

$$F = \frac{M}{K_y(y-y^*)} \int_{y_f}^{y_{in}} \frac{dy}{y-y^*} \quad (29)$$

or

$$M = K_y F \frac{y_{in} - y_f}{\int_{y_f}^{y_{in}} \frac{dy}{y - y^*}} \quad (30)$$

Comparing the equations (27) and (30)

$$K_y F \Delta y_m = K_y F \frac{y_{in} - y_f}{\int_{y_f}^{y_{in}} \frac{dy}{y - y^*}} \quad (31)$$

find the value of the average driving force

$$\Delta y_m = \frac{y_{in} - y_f}{\int_{y_f}^{y_{in}} \frac{dy}{y - y^*}} \quad (32)$$

When the driving force is expressed through the concentration of the substance to be distributed in the liquid phase x , the expression for the average driving force is obtained

$$\Delta x_m = \frac{x_f - x_{in}}{\int_{x_{in}}^{x_f} \frac{dx}{x^* - x}} \quad (33)$$

When the equilibrium line is straight ($y^* = mx$), the average driving force is defined as the average logarithmic force between the driving forces at the beginning and at the end of the phase contact surface:

$$\Delta y_m = \frac{\Delta y_{in} - \Delta y_f}{2,3 \lg \frac{\Delta y_{in}}{\Delta y_f}} \quad (34)$$

where $\Delta y_{in} = y_{in} - y_{in}^*$ – the driving force at the beginning of the phase contact surface;
 $\Delta y_f = y_f - y_f^*$ – the driving force at the end of the phase contact surface [2, 3].

Questions to control:

1. List the main mass-exchange processes.
2. What is the driving force of all mass-exchange processes?
3. Why are mass-exchange processes, as a rule, carried out in countercurrent devices?
4. Give the differential and integral equations of mass transfer for the entire surface of the phase contact.
5. What is the physical meaning of equation (11) of the working line of a continuous mass-exchange process.
6. Formulate the basic law of mass transfer.
7. What is the mass transfer coefficient?
8. Indicate all possible dimensions of the mass transfer coefficient.
9. By what equation is the average driving force of the mass transfer process calculated?
10. What happens to the mass-exchange process in establishing the equilibrium state?

Literature

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